|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | Min. | 1st Quartil | Median | Mean | 3rd Quartil | Max. |
| DEA radial model | 1.000 | 1.058 | 1.096 | 1.101 | 1.136 | 1.348 |
| SVM Polynomial | 0.005 | 1.035 | 1.075 | 1.072 | 1.115 | 1.305 |
| Neuronal Network | 0.885 | 1.045 | 1.075 | 1.082 | 1.115 | 1.335 |

In Table 1, we compare the results obtained by applying the radial model and machine learning (ML) models using our methodology. It is observed that, when detecting super-efficiency, the statistics of the SVM and neural network scores are generally lower than those of the radial model. The median and third quartile of the SVM and neural network scores are the same, although the neural network has a smaller interquartile range than the SVM, indicating less variability in the intermediate values. This is shown in Figure KERNEL, where the neural network scores are more concentrated around the mean, followed by SVM, and then the radial model. On the left tail, the scores estimated at 0.005 can be seen.

In terms of the mean scores, the results indicate that the scores by the radial model are, on average, higher compared to those obtained through machine learning techniques. Specifically, the scores of the radial model are, on average, 0.029 units higher than those of SVM Polynomial and 0.019 units higher than those of the Neural Network. This suggests that DEA tends to give higher efficiency scores compared to the other techniques analyzed.

The minimum score estimated by SVM is 0.005, significantly lower than any score observed in the other techniques. This low value is due to the fact that, during training, no proportional reduction of the outputs predicted a value lower than the threshold of 0.73. In particular, there are 5 DMUs for which the algorithm determined this extremely low score. Table 2 shows the scores of the DMUs classified as efficient by the radial model (40 DMUs) and their respective scores using the described ML techniques, as well as the radial model also detecting super-efficiency. In the latter, there are DMUs for which finding a score is infeasible.

A notable feature of using machine learning (ML) techniques to estimate efficiency scores is the ability to identify as inefficient observations that the radial model classifies as efficient, even when detecting super-efficiency. For example, this is evident in observations 9 and 28.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **id** | **DMU** | **DEA** | **SVM** | **NNET** | **sDEA** |
| 1 | 18 | 1 | 0,945 | 0,965 | 0,974 |
| 2 | 67 | 1 | 0,955 | 0,975 | 0,899 |
| 3 | 85 | 1 | 1,045 | 0,995 | - |
| 4 | 117 | 1 | 0,985 | 0,945 | 0,936 |
| 5 | 145 | 1 | 0,945 | 0,965 | 0,94 |
| 6 | 149 | 1 | 0,995 | 0,995 | 0,986 |
| 7 | 172 | 1 | 0,975 | 0,995 | 0,992 |
| 8 | 241 | 1 | 0,005 | 0,985 | - |
| 9 | 250 | 1 | 1,055 | 1,005 | 0,951 |
| 10 | 268 | 1 | 0,965 | 0,975 | 0,991 |
| 11 | 273 | 1 | 0,995 | 0,995 | 0,981 |
| 12 | 316 | 1 | 0,975 | 0,985 | 0,962 |
| 13 | 318 | 1 | 0,005 | 0,985 | - |
| 14 | 335 | 1 | 1,035 | 0,985 | - |
| 15 | 391 | 1 | 0,965 | 0,975 | 0,992 |
| 16 | 442 | 1 | 0,975 | 0,975 | 0,979 |
| 17 | 462 | 1 | 0,965 | 0,975 | 0,959 |
| 18 | 480 | 1 | 0,005 | 0,885 | - |
| 19 | 520 | 1 | 0,985 | 0,985 | 0,997 |
| 20 | 557 | 1 | 0,965 | 0,975 | 0,98 |
| 21 | 588 | 1 | 0,985 | 0,985 | 0,985 |
| 22 | 627 | 1 | 0,985 | 0,995 | 0,996 |
| 23 | 698 | 1 | 0,945 | 0,975 | 0,969 |
| 24 | 700 | 1 | 0,945 | 0,975 | 0,975 |
| 25 | 706 | 1 | 0,965 | 0,975 | 0,941 |
| 26 | 745 | 1 | 0,935 | 0,975 | 0,964 |
| 27 | 759 | 1 | 1,045 | 1,025 | - |
| 28 | 776 | 1 | 1,055 | 1,015 | 0,954 |
| 29 | 787 | 1 | 0,975 | 0,985 | 0,982 |
| 30 | 801 | 1 | 0,005 | 0,985 | - |
| 31 | 803 | 1 | 1,075 | 0,995 | - |
| 32 | 804 | 1 | 0,985 | 0,985 | 0,998 |
| 33 | 863 | 1 | 0,975 | 0,975 | 0,994 |
| 34 | 874 | 1 | 0,965 | 0,955 | 0,947 |
| 35 | 878 | 1 | 1,025 | 1,015 | 0,96 |
| 36 | 882 | 1 | 0,985 | 0,975 | 0,976 |
| 37 | 906 | 1 | 1,015 | 0,985 | 0,982 |
| 38 | 910 | 1 | 0,005 | 0,915 | 0,96 |
| 39 | 986 | 1 | 1,015 | 0,985 | 0,969 |
| 40 | 992 | 1 | 0,945 | 0,965 | 0,971 |